Light σ-Meson Production in Excited Υ Decay Processes II — Theoretical Investigation —

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The results of analyses obtained in Part I are examined from the viewpoint of chiral symmetry. The mass spectra of $\pi\pi$ system are widely believed to be suppressed generally near the threshold, because of the derivative coupling property of Nambu-Goldstone π meson. However, this suppression does not hold in the processes with large energy release, and the steep increase from the $\pi\pi$ threshold observed in the transition $\Upsilon(3S \to 1S)$ is shown to be consistent with general constraints from chiral symmetry.

§1. Introduction

In Part I,[1] we have analyzed systematically the $\pi\pi$ production amplitudes in the transitions, $\Upsilon(2S) \to \Upsilon(1S)$, $\Upsilon(3S) \to \Upsilon(1S)$, $\Upsilon(3S) \to \Upsilon(2S)$ and $\psi(2S) \to J/\psi(1S)$, and the $\pi\pi$ and KK production amplitudes in the trasition, $J/\psi \to \phi$. The production amplitudes $\mathcal{F}$ are parametrized in the form of the coherent sum of σ Breit-Wigner amplitude $\mathcal{F}_\sigma$ and direct 2π amplitude $\mathcal{F}_{2\pi}$, following the VMW method.

$$\mathcal{F} = \mathcal{F}_\sigma + \mathcal{F}_{2\pi}; \quad \mathcal{F}_\sigma = \frac{r_\sigma e^{i\theta_\sigma}}{m_\sigma^2 - s - i\sqrt{s}\Gamma_\sigma(s)}, \quad \mathcal{F}_{2\pi} = r_{2\pi} e^{i\theta_{2\pi}}. \qquad (1\cdot1)$$

Here r_σ ($r_{2\pi}$) is* production coupling constant of σ-state (2π-state) and $e^{i\theta_\sigma}$ ($e^{i\theta_{2\pi}}$) is a strong phase factor. The $m_{\pi\pi}$ or m_{KK} spectra are described well through all the relevant processes. Especially the double peak structure in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ decay spectra was nicely reproduced by the interference between $\mathcal{F}_\sigma$ and $\mathcal{F}_{2\pi}$. The obtained S-matrix pole position of σ state is $m_\sigma - i\Gamma_\sigma/2 = 526 - i150$MeV, which is taken commonly through all the relevant processes. This seems to give a strong evidence for existence of σ(500–600).

However, we must give special attention on the threshold behaviors. Because of the derivative coupling property of π meson as Nambu-Goldstone boson appearing in chiral symmetry breaking, the spectra of $|\mathcal{F}|^2$ is widely believed to be suppressed in the $\pi\pi$ low energy region. For example, in the $\pi\pi$ scattering, the observed spectra is suppressed near the threshold. In the linear σ model this suppression is produced by a strong cancellation between the σ amplitude and the $\lambda\phi^4$ amplitude. In the

* In our analysis the r_σ and $r_{2\pi}$ are taken as free parameters, since the $|\sigma\rangle$ state and $|2\pi\rangle$ state are considered, from quark physical picture,[7] as independent bases of S-matrix, and have independent production couplings, in principle.

relevant problem this suppression is actually observed experimentally in $\Upsilon(2S \to 1S)$ and $\psi(2S \to 1S)$, and is reproduced, similarly by the cancellation between $\mathcal{F}_\sigma$ and $\mathcal{F}_{2\pi}$. However, in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ decay, the steep increase from the $\pi\pi$ threshold is observed and is reproduced by constructive interference between $\mathcal{F}_\sigma$ and $\mathcal{F}_{2\pi}$, seemingly to be inconsistent with the derivative coupling property of pion.

The relevant decay processes had conventionally been treated as the intermediate two gluon emission process from transition between heavy quarkonium systems (described by the multipole expansion [2), 3)] of QCD), being accompanied by the conversion of gluons into pions (described by current algebra and PCAC). The main term of the amplitude, being proportional [3)] to s, comes from the trace of the gluonic part of QCD energy momentum tensor $\theta^G_{\mu\mu}$, which is enhanced due to the trace anomaly in QCD. The resulting amplitude has Adler 0 around $s \sim 0$, and predicts the suppression of the spectra near $\pi\pi$ threshold in the general processes, while this is not valid in the transition $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$, as was stated above. This seems to show that the multipole expansion method is* not applicable to the case with the larger energy release.

In the following we examine the consistency of our results, especially the threshold behavior of $\Upsilon(3S \to 1S)$, with constraints from chiral symmetry, and it is shown to be actually satisfied.

§2. Effective Lagrangian

2.1. *Suppression behavior in linear σ model*

First we study the threshold suppression of $\pi\pi$ scattering amplitude by SU(2) linear σ model (LσM). Through the mechanism of spontaneous chiral symmetry breaking the σ acquires a non-zero vacuum expectation value (VEV) $\sigma_0 = f_\pi$, and the $\sigma\pi\pi$ coupling ($\mathcal{L}_{\rm int} = -g_{\sigma\pi\pi}\sigma' \boldsymbol{\pi}^2$) appears. The $g_{\sigma\pi\pi}$ and λ (defined by $\mathcal{L}_{\rm int} = -\lambda(\boldsymbol{\phi}^2)^2/4$) are related with m_σ as

$$\sigma = \sigma_0 + \sigma';\ \sigma_0 = f_\pi.\ \ g_{\sigma\pi\pi} = f_\pi\lambda = (m_\sigma^2 - m_\pi^2)/(2f_\pi). \qquad (2\cdot1)$$

The $\pi\pi$ scattering $A(s,t,u)$ amplitude is given as the sum of terms, attractive σ amplitude and repulsive $\lambda\phi^4$ amplitude. According to Eq.(2·1), these two amplitudes strongly cancel with each other in $O(p^0)$ level(, where p means a momentum of the pion), leaving the $O(p^2)$ Tomozawa-Weinberg amplitude, which is consistent with the derivative coupling property of Nambu-Goldstone π meson.

$$A(s,t,u) = \frac{(-2g_{\sigma\pi\pi})^2}{m_\sigma^2 + (p_1+p_2)^2} - 2\lambda = \frac{1}{f_\pi^2}\left[\frac{(\cancel{m}_\sigma^2 - m_\pi^2)^2}{\cancel{m}_\sigma^2 + (p_1+p_2)^2} - (\cancel{m}_\sigma^2 - m_\pi^2)\right]$$

* Multipole expansion method is effective in case $\langle kr\rangle \ll 1$, where k is a typical momentum of the emitted gluon, which may be determined as $k \approx (M' - M)/2$ ($M'(M)$ being mass of the initial (final) quarkonium). r is the size of the quarkonium, which is estimated, by using the quark model, [4)] with the values, [5)] $\langle r\rangle = 3.5\mathrm{GeV}^{-1}$ for $\Upsilon(3S)$ and $\langle r\rangle = 2.3\mathrm{GeV}^{-1}$ for $\Upsilon(2S)$. Thus, $\langle kr\rangle \approx 0.65$ for $\Upsilon(2S \to 1S)$, while $\langle kr\rangle \approx 1.6$ for $\Upsilon(3S \to 1S)$. This fact suggests the expansion is not effective for $\Upsilon(3S \to 1S)$.

$$= \frac{(m_\sigma^2 - m_\pi^2)(-(p_1+p_2)^2 - m_\pi^2)}{m_\sigma^2 + (p_1+p_2)^2} \approx \frac{-(p_1+p_2)^2 - m_\pi^2}{f_\pi^2}, \qquad (2\cdot 2)$$

where p_1, p_2 are momenta of the emitted pions. The final form has Adler 0: The $A(s,t,u)$ vanishes when $p_{1\mu}$ is continued as $p_{1\mu} \longrightarrow 0_\mu$ but $p_{2\mu}$ remains on mass shell. This corresponds to zero at $s = -(p_1+p_2)^2 = m_\pi^2$, being close to the threshold.

2.2. *Effective Υ decay interaction–Non-derivative type*

The similar cancellation is obtained in the Υ decay amplitude derived by the effective chiral symmteric Lagrangian of non-derivative type,

$$\mathcal{L}_{\rm prod} = \xi_{2\pi} \Upsilon'_\mu \Upsilon_\mu (\sigma^2 + \boldsymbol{\pi}^2), \qquad (2\cdot 3)$$

where $\Upsilon'(\Upsilon)$ is the field of initial (final) $b\bar{b}$ quarkonium. $\xi_{2\pi}$ is the direct 2π (and 2σ) production coupling constant. Through the chiral symmetry breaking, the direct one-σ production coupling ($\mathcal{L}_\sigma = \xi_\sigma \Upsilon'_\mu \Upsilon_\mu \sigma$) appears, and the ξ_σ is related with $\xi_{2\pi}$.

$$\mathcal{L}_{\rm prod} = \xi_{2\pi} \Upsilon'_\mu \Upsilon_\mu (f_\pi^2 + 2 f_\pi \sigma' + \sigma'^2 + \boldsymbol{\pi}^2), \quad \xi_\sigma = 2 f_\pi \xi_{2\pi}. \qquad (2\cdot 4)$$

The $\pi\pi$ production amplitude $\mathcal{F}$ is given as the sum of $\mathcal{F}_\sigma$ and $\mathcal{F}_{2\pi}$, which cancel with each other in $O(p^0)$ level due to the constraint Eq. (2·4) of ξ_σ and $\xi_{2\pi}$,

$$\mathcal{F} = \mathcal{F}_\sigma + \mathcal{F}_{2\pi} = \frac{\xi_\sigma(-2g_{\sigma\pi\pi})}{m_\sigma^2 - s} + 2\xi_{2\pi} = 2\xi_{2\pi}\left(-\frac{\cancel{m}_\sigma^2 - m_\pi^2}{\cancel{m}_\sigma^2 - s} + \cancel{1}\right) = 2\xi_{2\pi}\frac{m_\pi^2 - s}{m_\sigma^2 - s},$$

where $s = -(p_1+p_2)^2$. The final amplitude takes $O(p^2)$ form, being consistent with the derivative coupling property of π meson. The Adler 0 occurrs at $s = -(p_1+p_2)^2 = m_\pi^2$. This is consistent with the experimental threshold behavior in $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$, while is not with that in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$.

2.3. *Effective Υ decay interaction–Derivative type*

In order to explain the threshold behavior of $\Upsilon(3S \to 1S)$ decay, we consider the following chiral symmteric "derivative type" interaction.*

$$\mathcal{L}^{(d)}_{\rm prod} = \xi^{(d)}_{2\pi} \partial_\lambda \Upsilon"_\mu \partial_\nu \Upsilon_\mu (\partial_\lambda \sigma \partial_\nu \sigma + \partial_\lambda \boldsymbol{\pi} \cdot \partial_\nu \boldsymbol{\pi}). \qquad (2\cdot 5)$$

Since this interaction is of derivative form, the mechanism of chiral symmetry breaking gives no one-σ production coupling. Thus, there is no $\mathcal{F}_\sigma$ amplitude cancelling $\mathcal{F}_{2\pi}$. Then the $\pi\pi$ production amplitude is given by

$$\mathcal{F}^{(d)} = -\xi^{(d)}(P" \cdot p_1 P \cdot p_2 + P" \cdot p_2 P \cdot p_1), \qquad (2\cdot 6)$$

where $P"(P)$ is the momentum of $\Upsilon(3S)(\Upsilon(1S))$.

In the relevant $\Upsilon(3S \to 1S)$ decay, the relation,

$$M_{\Upsilon(3S)} > M_{\Upsilon(1S)} \gg M_{\Upsilon(3S)} - M_{\Upsilon(1S)} (\equiv \Delta E) \gg m_\pi, \qquad (2\cdot 7)$$

* This interaction as well as Eq.(2·3) is easily shown to lead to the S-wave dominance of the $\pi\pi$ system, experimentally confirmed.[6]

holds, where $\Delta E(=895\mathrm{MeV})$ is the energy release of the relevant decay. Thus, in the rest frame of the initial $\Upsilon(3S)$ the final $\Upsilon(1S)$ is almost at rest, while the emitted two pion system accepts a large relativistic recoil velocity.

The $\mathcal{F}^{(d)}$ vanishes when $p_{1\mu} \to 0_\mu$. Thus this $\mathcal{F}^{(d)}$ has Adler 0, and satisfies the general constraints from chiral symmtery. However, the limit, $p_{1\mu} \to 0_\mu$, is far from the momentum in the physical $\pi\pi$ threshold, and the corresponding 0 does not lead to the suppression near the threshold, since at $s = 4m_\pi^2$ the pion four-momenta should be $p_{1\mu} = p_{2\mu}$ and the pion energy becomes $p_{10} = p_{20} \approx (M" - M)/2 = 450\mathrm{MeV} \gg 0$. Actually the $\mathcal{F}^{(d)}$ can be approximated as $\mathcal{F}^{(d)} \approx -2\xi^{(d)} M" M p_{10} p_{20}$, which is almost s-independent in all the physical region and has no zero close to the threshold. By using this type of amplitude we can explain the steep increase of the $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ spectra.

In the analysis of Part I we have used the free parameters r_σ and $r_{2\pi}$. They correspond to the ξ_σ and $\xi_{2\pi}$ in the effective Lagrangian, which have no constraints from chiral symmetry, and our treatment is proved to be correct.

§3. Comparison between situations of $\pi\pi$ production and scattering

Summarizing our considerations given in the previous sections, we compare the general features of $\pi\pi$ production processes with those of $\pi\pi$ scattering process in Table I.

Table I. Comparison of $\pi\pi$ production with $\pi\pi$ scattering

	$\pi\pi$ production	$\pi\pi$ scattering
Energy release ΔE Chiral momentum expansion	$\gg m_\pi$, generally large Not valid	≈ 0, close to threshold valid
Form of amplitude Amplitude near threshold	$P \cdot p_1 P \cdot p_2$ $\sim O(M\Delta E)^2$ large	$p_3 \cdot p_1$ etc. $\sim O(m_\pi^2)$ small
Cancellation of $\mathcal{F}_\sigma$ and $\mathcal{F}_{2\pi}$ near $\pi\pi$ threshold Adler 0 limit $p_{1\mu} \to 0_\mu$	generally No far from thres. region	Yes close to thres. region
Feature of spectra	Steep increase from threshold Direct σ peak in many cases	Suppresion near threshold No direct σ peak

The $\pi\pi$ production processes have generally much the larger ΔE than m_π. Thus, the momentum of emitted pion becomes large, and the chiral momentum expansion in nonlinear treatment of pion is not applicable. The derivative type amplitude, as Eq.(2·6), may play an important role. The Adler 0 exists but its limit $p_{1\mu} \to 0_\mu$ is far from the physical momentum in threshold region, and so it does not give the suppression at the small s region, the spectra shows steep increase from threshold, and the direct σ peak is observed in $m_{\pi\pi} \approx 500 \sim 600\mathrm{MeV}$.

On the other hand in the $\pi\pi$ scattering process, the $\Delta E \approx 0$ corresponds to the near-threshold region, and pion momentum itself becomes small. So the chiral perturbation theory is effective in this case, and the spectra are suppressed close to threshold.

We can see the above mentioned situations actually in the following examples:[8)]

In the pp-central collision experiment, $pp \to pp(\pi^0\pi^0)$, by GAMS the large event concentration in the low energy region $m_{\pi\pi} \approx 500$MeV is explained as due to direct production of σ resonance. The initial fast proton has large momentum of 450 GeV/c, and this process has very large ΔE giving the pion extremely high momentum in $\pi\pi$ threshold. In $J/\psi \to \omega\pi\pi$ decay, where ΔE is very large, the chiral cancellation does not occur, and the direct σ peak was observed directly.

In the relevant problem the ΔE are $(\Delta E_{\Upsilon(3S\to 1S)}, \Delta E_{\Upsilon(2S\to 1S)}, \Delta E_{\Upsilon(3S\to 2S)}; \Delta E_{\psi(2S\to 1S)}) = (895, 563, 332, 589)$MeV. Among these values the $\Delta E_{\Upsilon(3S\to 1S)}$ is the largest. Actually only this process shows the steep increase from threshold.

§4. Concluding Remarks

Through the above theoretical considerations we may conclude that the method of our analysis applied in Part I, is shown to be consistent with the general requirement from chiral symmetry. In the $\pi\pi$ production processes with large energy release to the $\pi\pi$ system, the chiral cancellation between σ amplitude and 2π amplitude near the $\pi\pi$ threshold does not occur, and the direct σ peak is generally observed, while in the $\pi\pi$ scattering with small energy release near the threshold the cancellation occurrs, and the direct σ peak cannot be observed in the spectra.

Finally we give a short comment on the possible origin of $\mathcal{L}^{(d)}$. If we have the coupling to intermediate tensor glueball states $G_{\mu\nu}$, $\mathcal{L}^G = \xi_g \partial_\lambda \Upsilon"_\mu \partial_\nu \Upsilon_\mu G_{\lambda\nu}$, the $\mathcal{L}^{(d)}$ is obtained. This form of $\mathcal{L}^G$ is naturally derived from the calculation in a covariant level classification scheme.[7)]

The other possible origin is the $B\bar{B}$ coupled channel effect,* $\Upsilon(3S) \to B\bar{B} \to \pi B^*\bar{B} \to \pi\pi B\bar{B} \to \pi\pi\Upsilon(1S)$, which is expected to be strong since the threshold energy $2m_B$ is very close to $m_{\Upsilon(3S)}$. If B^* is a scalar meson, the corresponding amplitude is proportional to the emitted pion energy, and is similar to Eq.(2·6).

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* The coupled channel amplitude, originally considered by Lipkin and Tuan,[9)] was used as the additional term to the multipole expansion amplitude by Moxhay,[10)] to explain the spectra of $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$. However, according to the theoretical estimate[11)] by using NRQM the contribution due to the $^3P_0$ scalar state is not suffucient to explain the experimental data. We consider the B^* should be taken as a chiral particle.[12)]